

# A Note on the Cut Benchmarks at Armagh Observatory: Marking Two Hundred Years of the Ordnance Survey of Ireland

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Abstract: Ordnance Survey (OS) benchmarks (OSBMs) dating from the first levelling of Ireland in the 1830s are important cultural artefacts which represent a key element of cartographic and mapping history worthy of protection. This note supplies images and GPS coordinates for the three surviving OSBMs in the grounds of the Armagh Observatory. The first, which we have previously used to determine the exact geographical location of the Observatory, is inscribed near the base of the south-east face of the Troughton telescope dome but has become difficult to find owing to spalling of the stonework on which it is formed. Unless efforts are made to secure its preservation it may soon be lost. The second is located at the foot of the western of the two gate posts defining the main entrance to the Observatory demesne close to the Gate Lodge. The third, which until recently had not been seen for a generation, was rediscovered in November 2021 at the base of the surviving stone pillar defining the entrance to the Observatory's original coach house and stables. A fourth OSBM was lost in the mid-1960s during construction of the Planetarium.

**Keywords:** historic buildings, Ordnance Survey, OS benchmarks, astronomy, geodesy, Armagh, Ireland

# 1. Colby's Survey

It is well known that astronomers' early efforts to map the heavens had an important practical application in determining the latitude and longitude of places on Earth. This was part of the work also of the astronomers at Armagh Observatory, notably the third director the Revd Dr John Thomas Romney Robinson (1793–1882), whose careful work in observing stellar positions during the first half of the nineteenth century led to publication of the famous 'Armagh Catalogue' (Robinson 1859) and to Robinson receiving the Gold (Royal) medal of the Royal Society in 1862.

Catalogues of accurate stellar positions contributed both to astronomy and the international project to enable seafarers accurately to measure positions at sea and cartographers to determine the exact latitude and longitude of features on maps, such as buildings, roads, county, city and field boundaries, and so on. During this time there was a close relationship between the sciences of astronomy and geodesy in measuring the size and exact shape of the Earth and the positions (*e.g.*, latitude, longitude and height) of points on its surface.

A key moment in this relationship occurred two hundred years ago when Lieutenant-Colonel Thomas Colby (1784–1852), Superintendent of the Trigonometrical Survey of the Board of Ordnance, was tasked by the UK Parliament in 1824 to carry out an accurate trigonometrical survey of Ireland. There are many sources describing how Colby and others approached this formidable task, the challenges they had to overcome, and how their pioneering efforts succeeded in producing the first accurate Ordnance Survey (OS) maps of the whole country (*e.g.*, Seymour 1980, Andrews 2006, Lilley 2017, 2020; Cory 2023).

Here we mark the two-hundredth anniversary of this work by considering the surviving evidence of Colby's survey in and around the grounds of Armagh Observatory. We focus on the cut benchmarks in the Observatory demesne because they provide a tangible link with the past and three of an original four can still be found. These OS benchmarks (OSBMs) are of particular interest because the Observatory, founded in 1789, was used to define the coordinate origin for the 1835 OS 6-inch maps of Co. Armagh (Andrews 2006, Appendix E; Bailey *et al.*, 2021) and because the main Grade A listed building is home to the oldest scientific institution in Northern Ireland, one of the leading astronomical observatories on the island of Ireland. The observatory's north and south meridian marks have been reviewed elsewhere (Butler 2016).

#### 1.1. County Maps

A major result of Colby's work was publication and dissemination of a complete set of 32 'six-inch' county maps of Ireland. Work relating to the survey began in 1824 with Colby and Lieutenant Thomas Drummond (1797–1840) traversing Ireland to select the most suitable hilltops and mountains to serve as principal trigonometrical stations. Under Colby's direction from September 1827 to November 1828 this was followed by measurement — with unprecedented accuracy — of the Lough Foyle Baseline (Cory 2023). The individual counties of Ireland were surveyed during the 1830s and early 1840s (Andrews 2006, Appendix G), Colby's work finally being completed in 1846 (Owen & Pilbeam 1992, Appendix, p.179).

Each county was divided into a number of rectangular plots, typically dozens but in some cases more than a hundred, with approximate dimensions  $32000 \times 21000$  feet (6×4 miles). At a scale of six inches to the mile the features within each rectangular plot in each county were drawn on sheets with sizes of approximately  $36 \times 24$  inches, the project to map the whole of Ireland requiring nearly 2000 sheets.

The Armagh Observatory's collection of early 6-inch OS maps, held in the Armagh Robinson Library, comprises 1904 sheets together with an index and cover sheet for each county. The library also holds a '60-inch' map of Dublin Castle and environs, and a later map of the City of Armagh at a scale of 1:500 divided into 16 sheets, surveyed in 1862 and engraved the following year.

The individual sheets for each county were arranged flush with one another on separate county grids albeit sometimes with imperfect alignment (Murphy & Ryan 1956), while the exact latitude and longitude of each county on the almost spherical surface of the Earth was fixed by a coordinate origin for each county.

These coordinate origins were usually identified with a prominent building or landmark (Andrews 2006, Appendix E, pp.325–326) on the basis that such a fiducial point would not move and that its geographic latitude and longitude with respect to Greenwich could be determined. Values for most counties were obtained by triangulation with respect to the principal nodes of Colby's survey, which had accurate positions using sightlines to high points in Wales, England and Scotland, in turn ultimately based on the Greenwich prime meridian.

Three cases used further sightlines to astronomical observatories, which also had accurately known positions tied to Greenwich. The coordinate origin for county Armagh was taken as coincident with the position of the observatory's earliest transit instruments, while for Dublin the origin was some 30 metres south-west of Dunsink's meridian room (Bailey *et al.*, 2021). The coordinate origin for County Sligo was Cooper's observatory at Markree (Andrews 2006; *cf.* Doberck 1884, p.331), approximately 1 m south and 14 m east of the observatory's transit instrument. In this way the exact locations of the coordinate origins for each of the 32 counties were ultimately constrained by astronomical observations tied to Greenwich.

Colby's survey set a high standard, and the 'six-inch' maps are still much in demand by people studying changes in the landscape over the last two hundred years and by historians and geographers investigating people's family histories and roots in the land and changes to Ireland's historic buildings and landscaped parklands and gardens. They provide a unique record of buildings, monuments, antiquities, place names, water courses *etc.*, as they were two hundred years ago.

#### 1.2. Benchmarks

Benchmarks provide an indication of the location and height of an object, often above mean sea level though from 8th April 1837, when it was selected and recorded, the zero of altitude on the island of Ireland was taken as the low water mark of the spring tide at Poolbeg lighthouse, Co. Dublin. This vertical datum, 'Poolbeg', remained in force in Ireland from 1837 to 1970.

As described by Andrews (Seymour 1980, p.88) and by Owen & Pilbeam (1992, p.42), Colby collaborated in 1842 with the mathematician and astronomer George Airy (1801–1892), Astronomer Royal (1835–1881), to obtain a three-month series of tidal observations at 22 points around the Irish coast. They discovered that mean

sea level, which Airy's research had found to be approximately 8 ft above Poolbeg, varied by up to 2.5 ft between stations compared with a range of 9 ft for the low water of spring tides.

This suggested that mean sea level was a more logical choice for the zero of altitude. However by this time Colby's survey had moved on apace and every county had been surveyed with heights relative to Poolbeg, and all except four engraved. At this point Colby the perfectionist (Cory 2023) decided to alter the Irish vertical datum but without changing the thousands of altitudes relative to Poolbeg already engraved. Soon afterwards, in 1848, his successor as Superintendent of the Ordnance Survey, Lieutenant-Colonel L.A. Hall, restored the Poolbeg vertical datum (Seymour 1980, p.149) and this choice remained in force until 1970 when it was agreed to change the datum to mean sea level at Malin Head determined from a 10-year series of observations at Portmore Pier from 1960 to 1969. The difference between the two vertical datums is approximately 2.71 m.

As further described by Andrews (Seymour 1980), benchmarks were originally permanent marks provided at intervals of one-third of a mile or less along a line of levelling. Owen & Pilbeam (1992, p.42) note that during the primary levelling from 1837 to 1843 approximately 2000 miles of forward and back levelling took place along the main Irish roads, results of which were added straight on to the early 6-inch maps. This suggests that at least 6000 benchmarks were shown on the early 6-inch maps with heights relative to Poolbeg. Many more were added in later years (and some removed) in response to changes in the built heritage, for example road widening or house building schemes, or loss of or changes to older buildings.

OSBMs come in various forms (Crowe 2020, Ordnance Survey 2022). The most common, the cut benchmark, was usually inscribed into the vertical brick or stone surface of a building, wall, or gatepost or another feature of the landscape, although some were carved into wooden posts. On more permanent structures, such as churches and bridges, the mark was sometimes augmented by a brass bolt, while in other cases it was inscribed on a horizontal surface such as a pavement or a window or door sill. In many such cases they have been lost during building or townscape renovations.

Another common type, the flush bracket, has the form of a metal plate set into a wall or the concrete pillar of one of the traditional OS triangulation points ('trig points') that can be seen on the tops of mountains and hills with commanding views of the countryside. These are readily identified by their unique serial number.

The complex network of OSBMs established by Colby and his successors gave coordinates and heights above Poolbeg of numerous features that could be used by surveyors as points from which to make local measurements. The heights of most benchmarks on the island of Ireland are still derived from these early measurements, although in some cases recalibrated by a fraction of a foot and with heights given in metres relative to Belfast or Malin Head. OSBMs thus have an important local mapping role. However, with the ready availability of hand-held GPS devices this professional use has become redundant, and the network is no longer maintained. In Britain, where there are more than 500,000 OSBMs in the OS database, no new ones have been created since 1993 and many are lost each year as roads change or buildings are demolished (Ordnance Survey 2018). A similar level of attrition applies to Northern Ireland, where despite there being 10,393 records in the OSNI OSBM database (OSNI OSBM Database 2023) no new ones have been added since possibly 1991 (Wilson & Wilson 2023) and others are being lost every year. For example, the OSBM shown at the top of Russell Street on Sheet 6 of the 1862 1:500 scale map of the City of Armagh was lost as recently as May 2009 during a pavement renovation scheme (Barden 2024). While this example was present in early May 2009, it is missing from smaller scale historic OS maps and the OSNI database. Conversely, there are some (*e.g.*, the OSBM on the door sill of the Church of Ireland Cathedral, Armagh) which exist but are also missing from the OSNI database.

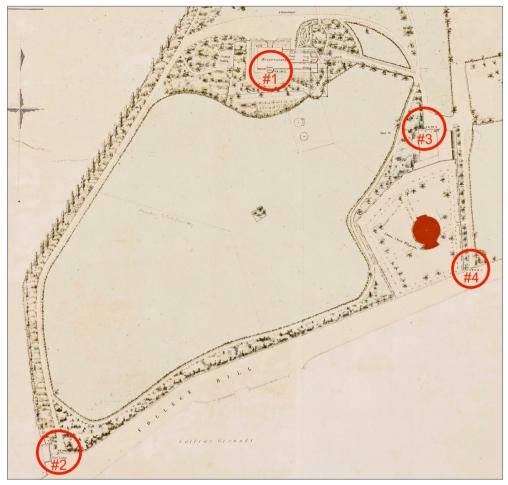
OSBMs play an important role in the history of map making and in the relationship between cartography and the physical environment. Although numerically decreasing, and often unappreciated, they represent an important part of the landscape with significant educational and heritage value. There is growing interest in discovering and describing OSBMs amongst academics, enthusiasts and local historians (*e.g.*, Bonsall 2020, Crowe 2020, Distel 2020, Wilson 2021, Wilson & Smith 2022, Wilson & Wilson 2023). Specialists and hobbyists alike make the point that as many of the surviving OSBMs as possible should be preserved for posterity.

Here we describe the three surviving cut benchmarks at Armagh Observatory, a Grade A listed building, each having a slightly different design and nowadays not all easy to find. They represent a significant fraction of the dozen or so benchmarks in the OSNI OSBM database within a kilometre of the city's Church of Ireland Cathedral. We hope that, by encouraging people to investigate these surviving elements of Colby's survey, this work will motivate efforts to maintain and preserve these important parts of Ireland's built heritage whilst contributing to improved understanding of the synergy between astronomy and geodesy two hundred years ago.

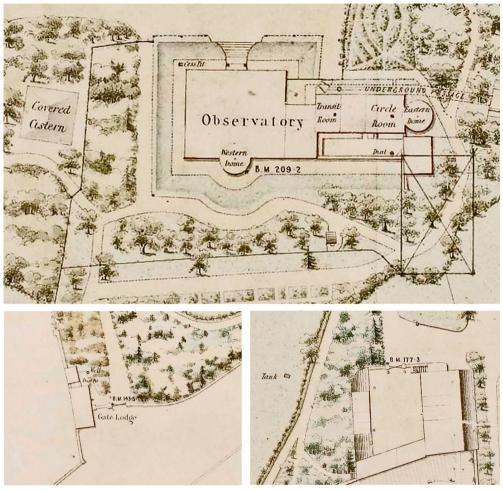
### 2. Armagh Observatory's Cut Benchmarks

There were originally four OSBMs in or close to the boundary of the Observatory demesne but only three survive. This section provides summary information and images for each of the surviving benchmarks as well as their GPS and Irish Grid coordinates.

Figure 1 shows part of a map of the Observatory demesne drawn around 1859. The numbered red circles show the positions of what then were four OS benchmarks though only one — that near the base of the Troughton dome — was included on Colby's original 1835 survey of county Armagh. The others are shown on later maps, for example the 1:500 Armagh City map surveyed in 1862.



**Figure 1.** The four OSBMs shown on an OS map of the Observatory demesne commissioned by the Governors and Guardians of the Observatory in 1859 and held in Armagh Observatory. North is to the top. Nowadays only three survive, namely those near (1) the base of the Troughton telescope dome on the main building; (2) the foot of the western gate post of the main entrance to the Observatory grounds from College Hill; and (3) the base of the surviving western gate pillar of the entrance to the original coach house and stables, near the footpath leading downhill from the Observatory to what is now the Planetarium. The fourth benchmark, on a gate post adjacent to the Observatory demesne close to the c.1900 Urban District boundary near what later became the main entrance to the Planetarium, was lost during construction of the Planetarium during the mid-1960s. The filled symbol identifies the position of the first Planetarium building. Further information is given in Figure 2.



**Figure 2.** Details of the surviving OSBMs shown in Figure 1. Top: OSBM #1 close to the base of the Troughton or West dome. At an elevation of 209.2 feet above the Poolbeg datum, itself 2.71 m below mean sea level, and 2.7 feet below ground level at the Observatory, this implies that the height of the Observatory above mean sea level at Belfast Lough is approximately 203.0 feet or  $H = 61.9 \pm 0.1$  m, two metres below that, H = 64 m, given in the Astronomical Almanac. Lower left: OSBM #2 at the main entrance to the Observatory grounds close to the Gate Lodge. Note the wall defining the western boundary of the Observatory demesne. Lower right: OSBM #3 on the surviving western gate pillar of the entrance to the Observatory's original coach house and stables. Source: OS map commissioned by the Governors and Guardians of the Observatory in 1859 and held in Armagh Observatory.



**Figure 3.** South elevation of Armagh Observatory showing the Troughton dome and location of the first OSBM to the right of the lower ground floor window of the dome and 2.7 feet below ground level. Image taken on 2009 December 21. Image credit Mark Bailey.

Figure 2 shows the positions of the three surviving OSBMs in more detail. The first and most important (OSBM #1) can be found near the base of the south-east face of the Troughton telescope dome. Figure 3 shows the south elevation of the observatory and the top of the basement window of the Troughton tower below which, and to the right, lies this first OSBM.

The following Figure 4 shows (top) the exact location of this OSBM #1, just to the right of the basement windowsill. It is now very difficult to find owing to spalling of the stonework on which it is formed and may soon be lost unless efforts are made to secure its preservation.

In a recent discussion of the buildings and location of Armagh Observatory, Bailey *et al.* (2021) showed how this first benchmark defines the exact position of Armagh Observatory on the Earth to an accuracy of a few tens of centimetres or approximately 0.01 arcseconds in latitude and longitude. (At the latitude of Armagh one-tenth of an arcsecond in longitude corresponds to approximately 1.806 m along a parallel, and one-tenth of an arcsecond in latitude corresponds to approximately 3.092 m on a meridian.)



**Figure 4.** Top left: Location of OSBM #1 to the right of the basement window and on the south-east face of the Observatory's Troughton telescope dome. Top right: Detail of this OSBM, image taken on 2016 April 21. Lower left: OSBM #2 at the foot of the western gate post at the entrance to the Observatory grounds near the Gate Lodge. Lower right: Detail of OSBM #2. Image credit: Mark Bailey.

The observation that the coordinate origin of the 1835 'six-inch' maps of county Armagh lies within half a metre of the position of the Observatory's earliest transit telescopes attests to the accuracy of the work of the astronomers and OS surveyors during the early nineteenth century.

The lower part of Figure 4 shows the second OSBM (#2), near the base of the western of the two gate posts at the entrance to the Observatory grounds near the Gate Lodge (Figures 1 and 2). This is easily found. However, despite an adjacent jostle or guard stone, providing a degree of protection to the pillar, it remains vulnerable to disturbance by collision with vehicles or by an occasionally expressed desire to widen the entrance to accommodate the larger service vehicles and lorries that sometimes use the entrance, which is now shared with the local Armstrong primary school.



**Figure 5.** Top: Location of the third surviving OSBM (#3) soon after its rediscovery in November 2021 at the foot of the surviving western gate pillar of the entrance to the Observatory's original coach house and stables. The historic wall (top left) leads west from the pillar before turning south. The top right image shows the north-east side of the pillar, which includes the OSBM engraved near its base close to ground level. Bottom: Detail of this OSBM shortly after rediscovery. Image credit: Mark Bailey

The third OSBM (#3) had lain unseen for at least a generation and was presumed lost. It was found by us in November 2021 hidden under dense foliage at the foot of the surviving western gate pillar of the entrance to the Observatory's original coach house and stables. It lies near the footpath leading downhill from the Observatory to what is now the Planetarium. The lower right image of Figure 2 shows the layout of the original coach house and stables adjacent to one of the Observatory's formal gardens (Figure 1). The context, detail and exact location of this OSBM is shown in Figure 5.

Table 1 presents the coordinates (GPS latitude and longitude, Irish Grid eastings and northings, and orthometric height) for the surviving OSBMs as well as the lost OSBM #4. The spatial uncertainty for the surviving OSBMs is estimated to be on the order of  $\pm$  0.1 m in each dimension, that for OSBM #4 is of the order of  $\pm$  1 m. We note (Bailey *et al.*, 2021) that conversion of these coordinates to any other relevant system can easily be accomplished online, for example using the OSI coordinate converter (see https://gnss.osi.ie/new-converter/) or the calculator created by Mark 'Tarquin' Wilton-Jones (see http://www.howtocreate.co.uk/php/gridref.php).

**Table 1.** Coordinates of the OSBMs in the Armagh Observatory demesne (Figure 1). Irish Grid (IG) eastings and northings and approximate latitudes and longitudes come from the list of OS benchmarks published by the Northern Ireland government (OSNI OSBM Database 2023; see link from https://admin.opendatani.gov.uk/dataset/osni-open-data-benchmarks-height). Height is orthometric height above mean sea level at Belfast Lough. IG coordinates for the fourth OSBM (#4) have been estimated from the position shown on the OSNI 6-inch Historical Third Edition (1900–1907) map available online from the PRONI Historical Maps Viewer.

Our ID	OS ID	GPS Latitude (N)			GPS Longitude (W)			Irish Grid		Height
		Deg	Min	Sec	Deg	Min	Sec	Easting (m)	Northing (m)	(m)
#1	8071	54	21	10.64	6	38	59.80	287829.309	345757.815	61.08
#2	403	54	21	3.57	6	39	6.62	287710.402	345537.075	41.05
#3	7393	54	21	9.57	6	38	54.92	287918.015	345726.612	51.35
#4	_	54	21	6.92	6	38	53.52	~287945	~345645	~46.4

## **3.** Conclusion

The three surviving OSBMs in the Observatory demesne highlight the connection between astronomy and Colby's creation of the first accurate maps of Ireland. They illustrate the accuracy of the work of the surveyors who, two hundred years ago, first mapped the city and surrounding county of Armagh.

The most important of the three OSBMs, namely #1, is that inscribed near the base of the south-east face of the Troughton dome. This is key for determining the exact location of the Observatory and related features in and around the Observatory demesne, such

as the earliest transit telescopes, telescope domes, and the surviving north and south meridian marks. This is the least well preserved of the three and is at risk of being lost.

The second OSBM (#2) is inscribed near the base of the western gate post at the entrance to the Observatory grounds. It requires minor repair but is otherwise easy to find.

The third (#3), recently revealed at the foot of the surviving western pillar of the entrance to the Observatory's original coach house and stables, is in a good state of repair having been obscured for at least the last 40 years. The fourth OSBM (#4), located close to what is now the entrance to the Planetarium, was lost in the 1960s during construction of a road entrance to the Planetarium.

The OSBMs discussed in this note illustrate their practical use, which was to record the exact height of points in the landscape and shown on maps, and an important stage in the development of cartography and the history of map making in Britain and Ireland. They also illustrate some of the ways by which OSBMs may be lost, for example by deterioration of the surface on which they are engraved; damage by passing traffic; obscuration by encroaching foliage; or destruction by road widening or urban pavement schemes. OSBMs are decreasing in number every year but are increasing in educational and cultural heritage value. We recommend that, wherever possible, surviving OSBMs should be preserved.

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### References

Andrews, J.H., 2006. A Paper Landscape: The Ordnance Survey in Nineteenth-Century Ireland. Second Edition 2002 reprinted 2006. Four Courts Press, Dublin, Ireland.

Bailey, M.E., C.J. Butler, J.A. Finnegan, S.T. Kelly, 2021. On the buildings and location of Armagh Observatory. *Irish Geography*, 54, No. 2, 109–140.

Barden, S., 2024. Personal Communication.

Bonsall., J., 2020. Take a look at these crows feet, just look: bench marks of Sligo. See https:// drjamesbonsall.com/2020/04/09/take-a-look-atthese-crows-feet-just-look-benchmarks-of-sligo/

Butler, C.J., 2016. Armagh Observatory's meridian marks. Astronomy and Geophysics, 57, 2.27–2.31.

**Cory, M.J., 2023.** *Mapping Monuments: The Lough Foyle Baseline.* The Mapping Monuments Project. Binevenagh and Coastal Lowlands Landscape Partnership Scheme and Queen's University Belfast.

Crowe, N.B., 2020. Benchmarks. Irish Community Archive Network (ICAN), National Museum of Ireland and Heritage Council. See https://www. ouririshheritage.org/content/archive/topics/ miscellaneous/benchmarks#\_edn12

Distel, A.-K., 2020. Benchmarks in Kilkenny. *Kilkenny Archaeological Society and Community Radio Kilkenny*. See https:// kilkennyarchaeologicalsociety.ie/benchmarks-inkilkenny/

Doberck, W., 1884. Markree Observatory. *Observatory*, 7, 283–288, and 329–332.

Lilley, K.D., 2017. Mapping the nation: landscapes of survey and the material cultures of the early Ordnance Survey in Britain and Ireland. *Landscapes*, 18, 178–199.

Lilley, K.D., 2020. Mapping Monuments: A Landscape Archaeology of the Ordnance Survey, Powerpoint Presentation, Queen's University Belfast. See https://mappingmonuments.s3.euwest-2.amazonaws.com/Mapping+Monuments+ lecture+by+K.+Lilley+15-12-20+powerpointcompressed.pdf

Murphy, T., Ryan, R., 1956. The latitudes and longitudes of the six-inch sheet maps of Ireland. Geophysical Bulletin No. 13, Dublin Institute for Advanced Studies, Dublin. Ordnance Survey, 2018. What is an OS Benchmark? See https://www.ordnancesurvey.co.uk/blog/25years-since-last-benchmark

Ordnance Survey, 2022. Benchmark or Trig Pillar: What's in a Name? See https://www. ordnancesurvey.co.uk/blog/benchmark-trigpillar-whats-name

OSNI OSBM Database, 2023. OSNI Open Data Benchmarks Height Datasets Open Data NI. File last updated May 2023. Available as .csv download from https://admin.opendatani.gov.uk/dataset/ osni-open-data-benchmarks-height

Owen, T., Pilbeam, E., 1992. Ordnance Survey: Map Makers to Britain since 1791. Ordnance Survey, Southampton. HMSO, London.

Robinson, T.R., 1859. Places of 5,345 Stars Observed from 1828 to 1854 at the Armagh Observatory with Dublin.

Seymour, W.A., (ed) 1980. A History of the Ordnance Survey. William Dawson & Sons, Folkestone, England. See https://www. ordnancesurvey.co.uk/documents/resources/oshistory.pdf

Wilson, P., 2021. Marks of the Ordnance Survey. Heritage Newsletter, Portrush Heritage Group. See https://pure.ulster.ac.uk/ws/portalfiles/ portal/92552192/Ordnance\_Survey\_marks.pdf

Wilson, P., 2022. Ordnance Survey markers around Coleraine. *The Bann Disc*, 28, 67–71. See https://pure.ulster.ac.uk/ws/portalfiles/ portal/108732509/2022\_Bann\_Disc.pdf

Wilson, P., Smith, A., 2022. Bench Marks and Flush Brackets in Keswick – a Legacy of the Ordnance Survey. *Transactions of the Cumberland and Westmoreland Antiquarian & Archaeological Society*, 22, 137–146. See https://pure.ulster. ac.uk/ws/portalfiles/portal/200563711/2022Or dnanceSurvey-Final.pdf

Wilson, P., Wilson, F., 2023. Benchmarks – a view from up north. Archaeology Ireland (Summer), 25– 29. See https://www.jstor.org/stable/27233734